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Honouring Günter Haring's life-long contributions

Network Protocol Performance Bounding Exploiting Properties of Infinite Dimensional Linear Equations

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A useful perspective from a workshop on past and recent contributions on PERFORMANCE (analysis and design)

Which is ***the*** approach after all these years of work?

Analysis ?

→ Computer-based simulations ?

→ Real experimentation?

A clear recent trend to the right ?   But ………………

The loops are important … in addition to own merit !!!

Some of the merits of Analysis

<u>Analysis helps figure out:</u>

- the underlying “physics and laws” of the system
- the impact on performance of (multiple) parameters, and the design space to be explored.
- stability region of a system
- coping with the curse of dimensionality through bounds and approximations
- the frequency of occurrence of rare events
- ……………………. etc …………………………………

Main Focus of this presentation

➢ Exploit renewal cycles and set up recursive equations capturing protocol and system dynamics

➢ Employ properties of solutions of IDLE (Infinite Dimensional Linear Equations) to derive bounds of key performance indices (stability, delay, etc)

➢ Exemplify the methodology by applying it the case of a Limited Sensing Random Access Protocol (LS-RAP)

➢ Outline other cases of applicability

The LS-RAP

Common channel shared by distributed, non-communicating users
Ternary feedback to the nodes at the end of the slot (I – C – S)
M-packets / message, 1 slot /packet, ω CNTR, sync'ed slot boundaries
B_0 : new active users (no attempt yet) – their CNTR=0
B_1 : new active users (to attempt in next slot) – their CNTR=1
….etc

<u>The algorithm:</u>

1) If $F = S$, then

a) $B_0 \to B_0$, b) $B_1 \begin{cases} \nearrow B_1 \text{ if } \omega > 0 \\ \searrow B \text{ if } \omega = 0 \end{cases}$, c) $B_k \to B_k$, $k \geq 2$.

2) If $F = C$, then

a) $B_0 \to B_1$, b) $B_k \to B_{k+1}$, $k \geq 2$

c) For each $b \in B_1$, $b \begin{cases} \nearrow B_2 \text{ with probability } 1-p \\ \searrow B_1 \text{ with probability } p \end{cases}$

The LS-RAP

The algorithm: (continued)

3) If $F = I$, then

a) $B_0 \to B_1$

b) If last nonidle slot was involved in a collision, then

For each $b \in B_2$, $b \begin{cases} \nearrow B_1 \text{ with probability } 1-p \\ \searrow B_2 \text{ with probability } p \end{cases}$,

$B_k \to B_k$, $k \geq 3$

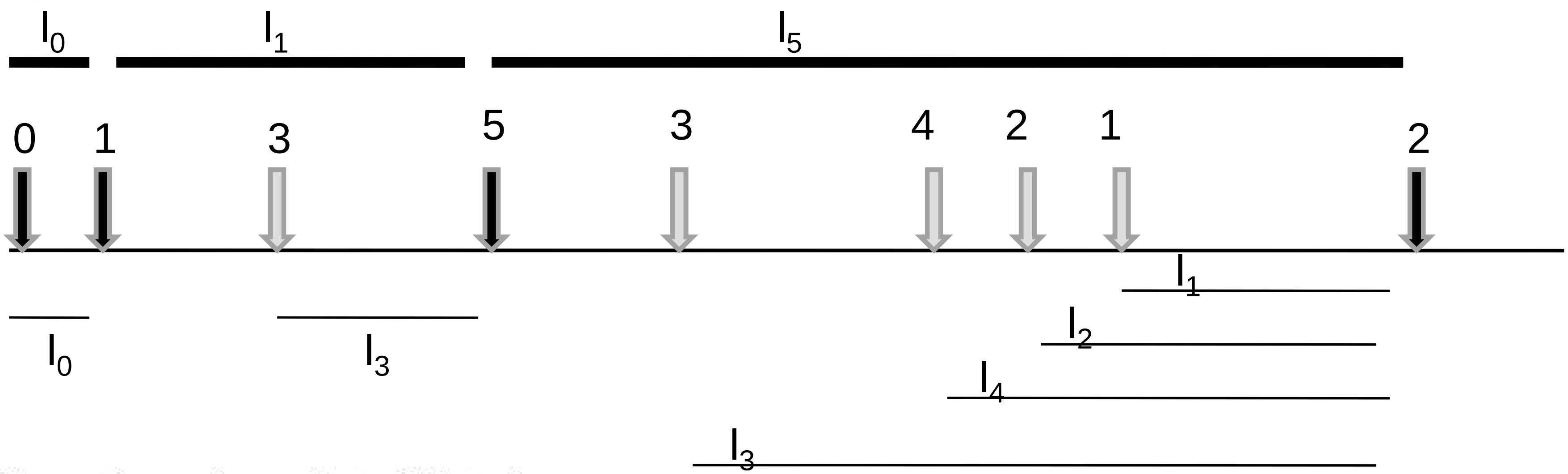
c) If last nonidle slot was involved in a successful transmission, then

c_i) if the current slot is the first idle slot after the successful one, then $B_k \to B_k$, $k \geq 2$

c_{ii}) if the current slot is not as in c_i), then $B_k \to B_{k-1}$, $k \geq 2$.

Recursive equations and IDLE

$l_k =$ Session of multiplicity k (collisions)



$$l_0 = 1, \qquad l_1 = 1 + M + l_{F_M}$$

$$l_k = 1 + \{1 + l_{k,0}\} I_{\{I_1+F_1=0\}} + \{l_{I_1+F_1} + l_{k-I_1+F_3}\} I_{\{I_1+F_1\neq 0\}} \ , k \geq 2$$

$$l_{k,0} = \{1 + l_{k,0}\} I_{\{I_2+F_2=0\}} + \{l_{I_1+F_1} + l_{k-I_1+F_4}\} I_{\{I_2+F_2\neq 0\}} \ , k \geq 2$$

Expectations yield:

$$L_k = h_k + \sum_{j=0}^{\infty} \alpha_{kj} L_j \ , k \geq 0 \ , \ h_k \geq 0, \ \alpha_{kj} \geq 0 \qquad (***)$$

Maximum Stable Throughput

$S_{\max}$ is derived by requiring the existence of a (finite for k finite) u.n.s. to (***)

<u>Def</u>: $S_{\max} = \max_{\lambda}\{\lambda : L_k < \infty, k < \infty\}$

<u>Prop 1:</u> If for some Λ^l there exist $L_k^u < \infty$, $k < \infty$,

$h_k + \sum \alpha_{kj} L_j^u \leq L_k^u$ then $L_k \leq L_k^u$

and $\Lambda^l \leq S_{max}$

($L_k^u = \beta(\Lambda,p)k - \gamma(\Lambda,p)$ was found analytically, will return)

<u>Prop 2:</u> If Λ_N^u is the max rate for which the truncated at N (***) has a u.n.s. then

$\Lambda_N^u \geq S_{max}$ and $\Lambda_N^u \xrightarrow[N \to \infty]{} S_{max}$

Delay Bounds

For $\lambda < \lambda^l$, $$D_S = \frac{C}{\lambda L}$$

$L=E\{L_k\}$, $C=E\{C_k\}$,

C_k = mean cumulative delay of messages over a session of multiplicity k (over L_k) – similar recursive eqns as for L_k - same h_k

Suffices to derive C^u, C^l, L^u, L^l

Upper Bounds on L_k, C_k

- $L_k^u = \beta(\lambda,p)k - \gamma(\lambda,p)$, $C_k^u = v_1 k^2 + v_2 k + v_3$ $k \geq 1$, $C_0^u = 0$

  Coefficients β, λ, v_1, v_2, v_3 analytically derived

  In line with linear / quadratic increase of L_k / C_k wrt k

  Improvement discussed later

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Lower Bounds on L_k, C_k

Def: Majorant/Minorant systems

If $A_k \geq |a_k|$, $B_{kj} \geq |b_{kj}|$, $0 \leq k \leq \infty$ then

$$x_k = A_k + \sum_{j=0}^{\infty} B_{kj} x_j \quad \text{is a majorant for} \quad y_k = a_k + \sum_{j=0}^{\infty} B_{kj} y_j$$

(MAJ) (MIN)

Theorem: If (MAJ) has nonnegative solution then (MIN) has solutions satisfying $|y_k| \leq x_k$

Since (***) has nonnegative solution for $\Lambda < \Lambda^l$ and N-truncated (***) is a minorant to (***) → $L_k^l \leq L_k$ (L_k^l : solutions to N-truncated (***))

Similarly $C_k^l \leq C_k$

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On Solution Complexity and Tightness of Bounds

Lower bounds

- Solution to N-truncated (***) is fast through successive substitutions. N can be very large, yielding tight lower bounds on L_k, C_k.

- Due to truncation effect at k=N (boundary effect):

  $L_{k_1}^{N_1} \approx L_{k_1}^{N_2}$ for $k_1 \ll \{N_1, N_2\}$ and $\ll N_1 \ll N_2$

  → $L^{N_1} \approx L^{N_2}$ since the Poisson multiplicities for small k contribute mostly to the mean L

- Tight upper bounds are more challenging

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On Solution Complexity and Tightness of Bounds

Upper bounds

- Derive some loose upper bounds, such as

  $L_k^u = \beta(\lambda,p)k - \gamma(\lambda,p), \quad C_k^u = v_1 k^2 + v_2 k + v_3$

- Formulate and solve the following N-truncated IDLE

$$x_k^{ut} = \underbrace{h_k + \sum_{j=N+1}^{\infty} a_{kj} x_j^{u}}_{h_k^t} + \sum_{j=0}^{N} a_{kj} x_j^{ut}$$

then $x_k \le x_k^{ut} \le x_k^{u}$, $0 \le k \le N$

where x_k solves $x_k^{ut} = h_k + \sum_{j=0}^{\infty} a_{kj} x_j \qquad (***)$

Tightening λ^u

1. λ^l (max λ yielding a u.n.s. to N-truncated (***)) does not change much as N increases, for N, even small (→ λ^l tight?)

2. $\lambda^l \ll \lambda^u$ → (from 1.), λ^u probably very loose

3. <u>Improving on λ^u</u> :

   $x_0^u = 1$, $x_k^u = (1+\varepsilon)L_k^l$, $1 \le k \le 7$

   $x_k^u = \beta(\lambda,p)k - \gamma(\lambda,p)$, $8 \le k \le \infty$ (as earlier)

   L_k^l is a lower bound from N-trancated (***)

   Not hard to select ε small s.t.

$$h_k + \sum a_{kj} x_j^{\,u} \le x_k^{\,u} \qquad (*)$$

   and then λ^{ut} is the max λ for which (*) has u.n.s.

Application to priority queuing

Assuming non-preemtive, work conserving disciplines

- Stability conditions known: $\rho = \rho_{FIFO} < 1$
- Mean renewal cycle length $X = X_{FIFO} = 1/(1-\rho)$
- Mean delay for class i: $D^i = C^i/(\lambda^i X)$

Lower bounds on D^i (D_{lo}^i)

Solving N-truncated IDLE for C_k

C_k = mean cumulative delay of packets served over a mean session X_k

X_k = mean length of the period between a time instant when the system is in state k, till the end of the renewal cycle

Application to priority queuing (cntd.)

Upper bounds on D^i (D_{up}^i)

Derived from the (usually known) FIFO equivalent result and the expression

$$D^{FIFO} = \sum_{i=1}^{k} \frac{\lambda^i}{\lambda} D^i$$

leading to

$$D_{up}{}^{i} = \frac{1}{\lambda^i}[\lambda D^{FIFO} - \sum_{k=1, k \neq i}^{k} \lambda^k D_{l_0}{}^{k}]$$

Conclusions of the presented work

- Exploit renewal cycles and set up recursive equations capturing protocol and system dynamics

- Employ properties of solutions of IDLE (Infinite Dimensional Linear Equations) to derive bounds of key performance indices (stability, delay, etc)

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