On traffic domination in communication networks

Walid Ben-Ameur[1] Pablo Pavon[2] Michał Pióro[3,4]

[1]TELECOM SudParis, France
[2]Technical University of Cartagena, Spain
[3]Warsaw University of Technology, Poland
[4]Lund University, Sweden

PERFORM Workshop 2010
Universität Wien, October 19, 2010

Outline

motivation

- communication network design problems frequently involve a large set of traffic matrices
    - multi-hour dimensioning
    - uncertain traffic
- a large subset of matrices is usually dominated by the rest
- identifying and deleting dominated matrices leads to problems with a reasonable number of matrices
    - increases computation efficiency
    - sometimes necessary to get a solution, especially in survivable network design

domination

- total domination:
  - matrix A *totally dominates* matrix B if for any link capacity reservation c and routing f that support A, c supports B using the same routing f
  - where routing f defines the split of a demand between the allowable paths (percentage of demand assigned to paths)
- ordinary domination:
  - matrix A *ordinarily dominates* matrix B if for any capacity reservation c that supports A using some routing f, c supports B, perhaps using different routing f'

Remark: total domination implies ordinary domination.

two known results

- assume a complete graph
- A totally dominates B iff $A \geq B$ component-wise
- A ordinarily dominates B iff B can be routed in a network with link capacity reservation A
- both results are due to Gianpaolo Oriolo

Remark: sufficiency is intuitively obvious.

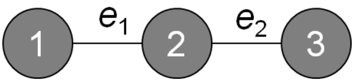
our results

- total domination - complete characterization
  - Oriolo's condition ($A \geq B$) is valid for 2-connected networks
  - for a general (connected) network, the same condition holds, but for a simple modification of matrices A and B ($A' \geq B'$)
  - generalization for a set of traffic matrices dominating a traffic matrix ($\mathcal{A}$ dominates B)
- ordinary domination
  - derivation of a necessary and sufficient condition in terms of a system of inequalities
  - giving evidence that checking for ordinary domination is $\mathcal{NP}$-hard

notation

- $G = (V, E)$ – graph
- V – set of nodes, $v \in V$
- E – set of undirected links , $e \in E$
- D – set of demands, $d \in D$
- $h = (h_d,\ d \in D)$ – traffic vector (instead of matrix A, B)
- P_d – set of all elementary paths for d, $P = \bigcup_{d \in D} P_d$
- $f = (f_p,\ p \in P)$ – flow (routing) vector
- $u = (u_e,\ e \in E)$ – link capacity reservation vector

example



$\hat{h} = (1, 0, 0), \ h = (0, 1, 1) \qquad D = \{13, 12, 23\}$

- clearly $\hat{h}$ dominates h both totally and ordinarily, and vice versa, still the Oriolo conditions are not satisfied
- in fact, both conditions are always sufficient but, as we can see, not necessary

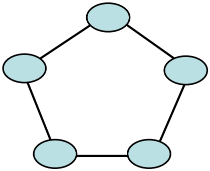
total domination - main result 1

Proposition 3

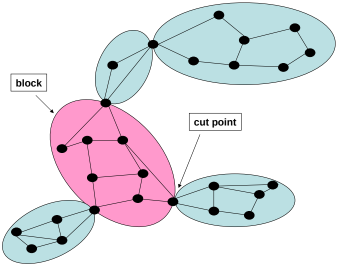
For 2-connected networks, $\hat{h}$ totally dominates h

if, and only if,

$\hat{h} \geq h$.



2-connected blocks and traffic augmentation



In each block we augment volumes h_d for the demands of the type: *cut point–cut point* and *cut point–inner point* by the volumes of transiting and terminating demands traversing the block. We treat each block as a separate network with such an augmented vector h^b.

total domination - main result 2

Proposition 4

For connected networks, $\hat{h}$ totally dominates h

if, and only if,

$\hat{h}^b \geq h^b$ in each block $b \in B$.

ordinary domination - main result

Proposition 5

Let $\pi = (\pi_e,\ e \in E)$, $\Pi = \{\pi :\ \pi \geq 0,\ \sum_{e \in E} \pi_e = 1\}$.

Then, $\hat{h}$ ordinarily dominates h if, and only if, for all $\pi \in \Pi$

$$\sum_{d \in D} \lambda_d(\pi)(\hat{h}_d - h_d) \geq 0,$$

where $\lambda_d(\pi)$ is the length of the shortest path for demand d.

a comment

Finding

$$\min_{\pi\in\Pi} \sum_{d\in D} \lambda_d(\pi)(\hat{h}_d - h_d)$$

is $\mathcal{NP}$-hard, suggesting that the condition in Proposition 5 is $\mathcal{NP}$-hard to check.

special case 1: $\hat{h}$ directly routeable in $G(V, E)$

Proposition 6

Suppose that for each $d \in D$ there exists a direct link $e(d)$ between the end nodes of demand d. Let $\hat{u}_{e(d)} = \hat{h}_d, \; d \in D$ and $u_e = 0$ otherwise.

Then, $\hat{h}$ ordinarily dominates h if, and only if, $\hat{u}$ supports h.

special case 2: ring networks

- $V = \{v_0, v_1, ..., v_{n-1}\}, E = \{e_0, e_1, ..., e_{n-1}\}$
- $e_i = v_i v_{i+1} \pmod{n}, i = 1, 2, ..., n-1$
- $\{e_i, e_j\}$ – cut, $h(e_i, e_j)$ load induced by h on the cut

Proposition 7

$\hat{h}$ ordinarily dominates h if, and only if,

$$\forall\, 0 \leq i, j < n,\ \hat{h}(e_i, e_j) \geq h(e_i, e_j).$$

- Easy to check.

conclusions

- complete, simple to check result for total domination (useful)
- complete result for ordinary domination (probably $\mathcal{NP}$-hard to check but can be useful in practice)

literature (strictly related to this paper)

- G. Oriolo: Domination between traffic matrices, *Mathematics of Operations Research*, vol.33, no.1, pp. 91–96, 2008.
- P. Pavon-Mariño and M. Pióro: On total traffic domination in non-complete graphs, submitted to *Operations Research Letters*, 2010.